International Journal of Theoretical Physics, Vol. 31, No. 2, 1992

Anomalous e^+e^- Pairs in Heavy Ion Collisions and Solar Neutrinos

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Received May 13, 1991

The production of anomalous e^+e^- pairs in heavy ion collisions and the solar neutrino puzzle are two seemingly unrelated problems of the standard model of electroweak interactions. According to the observations made at Homestake and Kamiokande, the flux of solar neutrinos is too small. Furthermore, the observations made at Homestake (neutrino–nucleon scattering) show anticorrelation of the solar neutrino flux with sunspots, unlike the observations made in Kamiokande (neutrino–electron scattering). According to the previously proposed model inspired by T(opological) G(eometro) D(ynamics), anomalous e^+e^- pairs result from the decay of the leptopion, which can be regarded as a bound state of color excited electrons. In this paper we show that the generalization of PCAC ideas leads to a prediction for the lifetime and production cross section of the leptopion in agreement with data. The model is also consistent with constraints coming from Babbha scattering and supernova physics. Leptopion exchange implies a new weak interaction between leptons at low cm energies (of the order of a few MeVs), which explains the Kamiokande–Homestake puzzle. Part of the solar neutrinos are transformed in the convective zone of the Sun to right-handed neutrinos inert with respect to ordinary electroweak interactions, but interacting with electrons via leptopion exchange so that they are observed in Kamiokande. A correct average value for the neutrino flux at Kamiokande is predicted using as input the Homestake flux, and the anticorrelation with sunspots in Kamiokande is predicted to be considerably weaker than in Homestake.

1. LEPTOPION HYPOTHESIS

The *solar neutrino problem* (Davis, 1986) and the *anomalous e^+e^- production in heavy ion collisions* (Chodos, 1987) are indications that something might be wrong with the standard model of electroweak interactions.

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0020-7748/92/0200-0229$06.50/0

The recent experimental situation concerning the solar neutrino problem is rather puzzling:

(a) According to the observations made at Homestake (Davis, 1986; Davis *et al.*, 1988), the observed flux of neutrinos from the Sun is considerably smaller than the theoretical estimates and shows *anticorrelation with the number of sunspots.*

(b) The recent observations from Kamiokande (Hirata *et al.*, 1986) are consistent with *the absence of anticorrelation*(!), although the experimental uncertainties are quite large. A crucial observation concerning the possible explanations of the (possibly real) Homestake–Kamiokande puzzle is that the detection of neutrinos in *Homestake* is based on *neutrino–nucleon* scattering, whereas in *Kamiokande, neutrino–electron* scattering is used.

The simplest models explaining the anticorrelation with sunspots assume that neutrinos possess magnetic moment (Voloshin *et al.*, 1986; Fugusita and Tanagida, 1987), so that left-handed neutrinos are transformed to right-handed neutrinos in the magnetic field of the solar spot: if the right-handed neutrinos have no weak interactions, they remain unobserved. These models have, however, some difficulties. First, it is not easy to explain the needed large magnetic moment in ordinary gauge theory models. Second, the large magnetic moment implies potential difficulties with supernova physics (Hirata *et al.*, 1987). The large chirality flip rate of left-handed neutrinos to inert right-handed neutrinos implies the runaway of neutrinos from a supernova in a time smaller than the time of order 10 sec observed for SN1987A (Hirata *et al.*, 1987). Third, these models (as such) do not solve the Kamiokande–Homestake puzzle.

The production of anomalous e^+e^- pairs observed in heavy ion collisions (Clemente *et al.*, 1984; Cowan *et al.*, 1985; Kraus and Zeller, 1986; Tsertos *et al.*, 1985) at energies just needed to overcome the Coulomb wall has been difficult to explain in terms of standard physics (Chodos, 1987). The explanation of the pairs as annihilation products of resonances having mass in the range 1.026–1.8 MeV and lifetimes of the order 10^{-10} sec seems to be the most natural explanation (Chodos, 1987). For example, the explanation of the e^+e^- bound states as an indication of a strongly interacting phase of QED has been proposed (Chodos, 1987).

In the following we shall propose a common solution to these two puzzles based on a *topological geometrodynamics* (Pitkänen, 1981, 1985, 1986*a,b*, 1988, 1990*a,b*) approach to the unification of basic interactions. The basic idea of this approach is that *space-time is representable as a submanifold* of the space $M^4_+ \times CP_2$, where CP_2 is the complex projective space of two complex dimensions (Pope, 1980; Gibbons and Pope, 1977; Pitkänen, 1990*b*) and M^4_+ denotes the future light cone of Minkowski ($m_{kl}k^k m^l \geq 0$). The isometry group of CP_2 is $SU(3)$ and is identified as the color group and the holonomy group associated with the generalized spinor structure of CP_2 is $SU(2)_L \times U(1)$ and is identified as the electroweak gauge group.

The explanation of the anomalous e^+e^- production in terms of the *leptopion hypothesis* (Pitkänen, 1990*a*) is based on the following ideas.

(a) One of the basic differences between TGD and QCD color is that TGD color is not a spinlike quantum number: colored states corresponds to partial waves in CP_2. As a consequence, TGD predicts the existence of color excited states of leptons and their color bound states and therefore a new branch of physics, which might be called leptohadron physics.

(b) The TGD-inspired explanation for e^+e^- resonances (Pitkänen, 1990*a*) is as *leptopions, pseudoscalar bound states of color excited leptons*. In Pitkänen (1990*a*) a model for the production of leptopions was proposed. Generalizing well-known ideas about low-energy pion physics, the leptopion was identified as the instanton density of the electromagnetic field proportional to the inner product $E \cdot B$ of electric and magnetic fields. The production amplitude for the leptopion was identified essentially as the Fourier transform of the quantity $E \cdot B$ associated with the system of two colliding heavy nuclei.

In the following we shall develop the leptopion hypothesis further and show that the *generalization of PCAC arguments* (Okun, 1982, p. 34) to the leptonic sector (a) predicts the existence of the leptopion and a *correct order of magnitude for the leptopion lifetime* and *production cross section in heavy ion collisions*, and (b) predicts a *new weak interaction between leptons based on leptopion exchange*. At cm energies of the order of the leptopion mass (1 MeV) the strength of this interaction is comparable to the strength of the ordinary weak interactions, but at high energies the interaction becomes unobservably weak, so that no new effects are predicted at high energies. Since the parameters of the standard model are determined from high-energy weak interactions, no changes are predicted to the parameters of the standard model. In particular, the lifetime of the leptopion is so long that the resonance is not observed in Babbha scattering at present energies. Further, (c) the new interaction *transforms right-handed neutrinos to electrons* and vice versa and is effective at cm energies not much larger than 1 MeV and can be *comparable to* or even *dominate over the ordinary weak interaction if the mass of the leptopion is sufficiently small*. It is this feature which might explain the Homestake–Kamiokande puzzle. Since the solar neutrinos have energy of the order of 5 MeV, their cm energy is of the order of 1 MeV and therefore the interaction of right-handed solar neutrinos with electrons is comparable to the ordinary weak interactions, so that they become *detected at Kamiokande, but not in Homestake* and *anticorrelation with sunspots gets weaker in Kamiokande* and vanishes if leptopion exchange dominates ($\sigma_R \simeq \sigma_L$). It turns out that if the lightest leptopion has mass $m \simeq 1.062$ MeV and about 1.05 times the predicted coupling to leptons, a *correct value for the average neutrino flux in Kamiokande* is predicted and the *anticorrelation* is predicted to be *considerably weaker than in Homestake.*

The second problem is to explain the *anticorrelation of the neutrino flux with sunspots* (Davis *et al.*, 1988) and at this stage further TGD-based ideas are needed and we leave the discussion of the problem to a separate paper.

2. LEPTOPIONS FROM GENERALIZED PCAC HYPOTHESIS

One can say that the PCAC hypothesis predicts the existence of the pion and a connection between the pion–nucleon coupling strength and the pion decay rate to leptons. In the following we give the PCAC argument and its generalization and consider various consequences.

2.1. PCAC for Ordinary Pions

The PCAC argument for ordinary pions goes as follows (Okun, 1982):

(a) Consider the contribution of the hadronic axial current to the matrix element describing lepton–nucleon scattering (say $N+\nu \rightarrow P+e^-$) by weak interactions. The contribution in question reduces to the well-known current–current form

$$\begin{aligned} M &= (G/\sqrt{2}) g_A L_\alpha \langle P|A^\alpha|N\rangle \\ L_\alpha &= \bar{e}\gamma_\alpha(1+\gamma_5)\nu \\ \langle P|A^\alpha|N\rangle &= \bar{P}\gamma^\alpha\gamma_5 N \end{aligned} \tag{1}$$

where $G=\pi\alpha/2m_W^2\sin^2(\theta_W)$ denotes the dimensional weak interaction coupling strength and g_A is the nucleon axial form factor: $g_A \simeq 1.253$.

(b) The matrix element of the hadronic axial current is not divergenceless, due to the nonvanishing nucleon mass,

$$q_\alpha\langle P|A^\alpha|N\rangle \simeq 2m_p\bar{P}\gamma_5 N \tag{2}$$

(Here q^α denotes the momentum transfer vector.) In order to obtain divergenceless current, one must modify the expression for the matrix element of the axial current

$$\langle P|A^\alpha|N\rangle \rightarrow \langle P|A^\alpha|N\rangle - q^\alpha 2m_p\bar{P}\gamma_5 N/q^2 \tag{3}$$

(c) The modification introduces a new term to the lepton–hadron scattering amplitude identifiable as an exchange of a massless pseudoscalar particle:

$$\delta T = (Gg_A/\sqrt{2})L_\alpha(1/q^2)q^\alpha 2m_p\bar{P}\gamma_5 N \tag{4}$$

The amplitude is identifiable as the amplitude describing the exchange of the pion, which gets its mass via the breaking of chiral invariance, and one obtains by the straightforward replacement $q^2 \to q^2 - m_\pi^2$ the correct form of the amplitude.

(d) The nontrivial point is that the interpretation as pion exchange is indeed possible since the amplitude obtained is to a good approximation identical to that obtained from the Feynman diagram describing pion exchange, where the pion–nucleon coupling constant and pion decay amplitude appear

$$T_2 = (G/\sqrt{2}) f_\pi q^\alpha L_\alpha (1/(q^2 - m_\pi^2)) g\sqrt{2}\, \bar{P}\gamma_5 N \tag{5}$$

This follows from the Goldberger–Treiman relation (Okun, 1982)

$$g_A\ (\simeq 1.25) \simeq \sqrt{2} f_\pi g/2m_p\ (\simeq 1.3) \tag{6}$$

2.2. PCAC in Leptonic Sector

A natural question is why not *generalize the previous argument to the leptonic sector* and look at what one obtains, and this is what we shall do next.

(a) Consider the contribution of the leptonic axial current to the scattering of leptons by weak interactions. The standard contribution is given by the expression

$$\begin{aligned} T &= (G/\sqrt{2}) \langle L_1 | A^\alpha | L_2 \rangle \langle L_3 | A_\alpha | L_4 \rangle \\ \langle L_i | A^\alpha | L_j \rangle &= \bar{L}_i \gamma^\alpha \gamma_5 L_j \end{aligned} \tag{7}$$

(b) As before, the matrix element of the axial current between leptonic states is not divergenceless,

$$q_\alpha \langle L_i | A^\alpha | L_j \rangle = [m(L_i) + m(L_j)] \bar{L}_i \gamma_5 L_j \tag{8}$$

and the divergenceless current is obtained by the replacement

$$\langle L_i | A^\alpha | L_j \rangle \to \langle L_i | A^\alpha | L_j \rangle - [m(L_i) + m(L_j)] q^\alpha \bar{L}_i \gamma_5 L_j / q^2 \tag{9}$$

(c) The replacement introduces a new term into the lepton–lepton scattering amplitude given by the expression

$$\delta T = -(\sqrt{2}\, G)[m(L_1) + m(L_2)] \bar{L}_1 \gamma_5 L_2 (1/q^2) [m(L_3) + m(L_4)] \bar{L}_3 \gamma_5 L_4 \tag{10}$$

The natural interpretation of this amplitude is as the exchange of a pseudoscalar particle, which becomes massive by the breaking of chiral invariance so that the replacement $q^2 \to q^2 - m_{\pi_L}^2$ should be made.

In accordance with the leptopion hypothesis introduced earlier (Pitkänen, 1990*a*), to explain the anomalous lepton pair production in heavy ion collisions we interpret the pseudoscalar particle as a "leptopion" having mass of the order of $m_{\pi_L} \simeq 1.062$ MeV. Of course, there are also higher excited states with quantum numbers of the leptopion and a natural expectation is that apart from numerical factors of order one, the same amplitude describes the interaction mediated by these states.

2.3. Consequences

The PCAC argument makes it possible to predict the leptopion coupling, leptopion lifetime, and production cross section, as the following arguments show.

(a) The value of the dimensionless lepton–leptopion coupling g (analogous to nucleon–pion coupling) is given by the expression

$$g(L_1, L_2) = \sqrt{G}\, 2^{1/4}[m(L_1) + m(L_2)] \tag{11}$$

The order of magnitude of this coupling is $g(e, e) \equiv g \simeq 2.8 \times 10^{-6}$. It is perhaps worth noticing that the value of the coupling constant is of the same order as the lepton–Higgs coupling constant and also proportional to the mass of the lepton. This in accordance with the idea that the components of the Higgs boson correspond to the divergences of various vector currents (Pitkänen, 1990*b*).

(b) The value of the coupling can be used to predict the *decay rate of the leptopion to leptons* and one obtains for the decay rate $\pi_L^0 \to e^+e^-$ the estimate

$$\begin{aligned} \Gamma &= [g(e, e)^2/4\pi] m_{\pi_L} (1 - 4m_e^2/m_{\pi_L}^2)^{3/2} \\ &= G m_e^2 (\sqrt{2}/\pi)(1 - 4m_e^2/m_{\pi_L}^2)^{3/2} m_{\pi_L} \end{aligned} \tag{12}$$

for the decay rate of the leptopion: substituting the mass of the lowest leptopion $m_{\pi_L} \simeq 1.062$ MeV, one obtains for the decay rate the estimate $\Gamma \simeq (1/2.7 \times 10^{-8})/\text{sec}$: the low decay rate is due to the phase space suppression, which gives a factor of order 10^{-2} to the decay rate. There are good reasons to expect that the same kind of estimate holds true for the lifetimes of higher mass excitations of the leptopion and for these states lifetimes are of the order of $\tau \simeq 10^{-10}$ sec. These values are *in accordance with the estimates of the lifetime obtained from heavy ion collisions* (Koenig *et al.*, 1987)!

The large value of the lifetime is also in *accordance with the limits for the lifetime obtained from Babbha scattering* (Judge *et al.*, 1990), which indicate that the lifetime must be longer than 10^{-12} sec.

(c) One can also estimate the *decay rate of the leptopion to two-photon final states*:

(i) As in the case of the ordinary pion anomaly, considerations give the following expression for the decay rate of the leptopion to two-photon final states:

$$\Gamma(\pi_L \to \gamma\gamma) = \alpha^2 m_{\pi L}^3/(64 f_{\pi L}^2 \pi^3) = (m_{\pi L}/m_\pi)^3 (f_\pi/f_{\pi L})^2 \Gamma(\pi \to \gamma\gamma) \quad (13)$$

where the decay rate for the ordinary pion is given by $\Gamma(\pi \to \gamma\gamma) \simeq 1.5 \times 10^{-16}$ sec.

(ii) As in the case of the pion, the value of the parameter f_{π_L} is defined in terms of the matrix element of the axial current between the pion and vacuum,

$$\langle \text{vac}|A_\alpha|\pi_L\rangle = ip_\alpha f_{\pi_L} \quad (14)$$

which implies for the decay rate of the leptopion the expression

$$\Gamma(\pi_L \to e\nu_e) = [G^2 m_e^2 f_{\pi L}^2 (1-x^2)^2/4\pi] m_{\pi_L}$$
$$x = m_e/m_{\pi_L} \quad (15)$$

(iii) On the other hand, the expression for the decay rate is obtained by a similar calculation as for the decay rate of the neutral leptopion,

$$\Gamma(\pi_L \to e\nu_e) = (Gm_e^2\sqrt{2}/8\pi)(1-2x^2)(1-4x^2)^{1/2} m_{\pi_L} \quad (16)$$

Comparing two expressions for the decay rate, one obtains

$$f_{\pi_L}^2 = [(1-2x^2)(1-4x^2)^{1/2}/(1-x^2)^2]/G$$
$$x = m_e/m_{\pi_L} \quad (17)$$

The value is large and of quite a different order of magnitude than the naive estimate $f_{\pi_L} \simeq m_{\pi_L}$ represented in Pitkänen (1990*a*). The expression for the decay rate

$$\Gamma(\pi_L \to \gamma\gamma) = (\alpha^2 G m_{\pi L}^2/64\sqrt{2}\,\pi^3)[(1-x^2)/(1-2x^2)(1-4x^2)^{1/2}] m_{\pi_L} \quad (18)$$

The decay rate is very small and implies the branching ratio $\Gamma(\pi_L \to \gamma\gamma)/\Gamma(\pi_L \to e^+e^-) \simeq 10^{-4}$ in the case of the lightest leptopion. Experimentally no evidence for other decay modes than $\pi_L \to e^+e^-$ has been found and the empirical upper limit for the $\gamma\gamma/e^+e^-$ branching ratio (Danzmann *et al.*, 1989) is $\Gamma(\gamma\gamma)/\Gamma(e^+e^-) < 10^{-3}$.

It should be noticed that the extremely low value of the decay rate implies that *it is not possible to exclude the possible existence of leptopion states having masses lower than* $2m_e$. These states would be stable against the decay to lepton pairs and have a lifetime of the order of 10^{-4} sec and therefore are impossible to detect if produced in heavy ion collisions: the upper limit posed on the lifetime by the size of the experimental apparatus is of the order of 10^{-9} sec (Chodos, 1987).

(d) The leptopion hypothesis predicts also the *production rate of the leptopion in heavy ion collisions* and the *correct order of magnitude* is predicted under rather mild assumptions.

In Pitkänen (1990*a*) we calculated only the differential probability distribution d^3P/dp^3 for the production of leptopions in the collision of heavy nuclei with given impact parameter and demonstrated that the properties explain the peculiar production characteristics of leptopions (leptopions are apparently produced at rest in cm coordinates). The assumptions of the model are the following:

(i) The *vacuum expectation value of the leptopion field* in the electromagnetic fields of colliding nuclei is *proportional to the instanton density of the electromagnetic field of the colliding nuclei,*

$$\begin{aligned} \langle \text{vac}|\pi_L(x)\rangle &= K\bar{E}(x)\cdot\bar{B}(x) \\ K &= \alpha/(8\pi m_{\pi_L}^2 f_{\pi_L}) \end{aligned} \tag{19}$$

where $\bar{E}$ and $\bar{B}$ are the electric and magnetic fields of the colliding nuclei. This idea is a direct generalization of the *anomaly considerations used to estimate the decay rate of the ordinary pion to two photons.* The decay rate is considerable since the *electric and magnetic fields of different colliding nuclei are large and nonorthogonal.*

(ii) The probability amplitude for producing leptopions in the collision of two nuclei is assumed to be given by the expression

$$\begin{aligned} A(p) &= \int f_p(x)\,\Box\langle\pi_L|\text{vac}\rangle\, d^4x \\ f_p &= e^{ip\cdot x}/(4\pi EV)^{3/2} \end{aligned} \tag{20}$$

The formula is obtained by generalizing the *LSZ formula.*

The *differential production probability* is obtained as

$$\begin{aligned} d^3P &= (\alpha/8\pi f_{\pi_L})^2|U|^2\, d^3p/2E_p \\ U &= \int e^{ip\cdot x}\bar{E}\cdot\bar{B}\, d^4x \end{aligned} \tag{21}$$

The Fourier component of $\bar{E}\cdot\bar{B}$ was calculated in Pitkänen (1990*a*) and it was found that it diverges when the leptopion is produced at rest in velocity cm coordinates, but no estimate for the production cross section was performed.

(iii) The *differential cross section for the leptopion production* is obtained by integrating the production probability over impact parameter b and azimutal angle Φ describing the position of the incoming nucleon in the plane normal to the momentum of the incoming nucleon,

$$d^3\sigma/dp^3 = \int [d^3P(b)/dp^3] b\, db\, d\Phi \tag{22}$$

It has turned out that the *integral over impact parameters diverges* (in analogy with the ordinary Rutherford cross section) and one must pose a cutoff in the b integration. A *natural cutoff* is given by the *interatomic distance a of the target nuclei*: $a \simeq 10^{-10}$ m.

(iv) The *total production cross section* can be written in the form

$$\begin{aligned} \sigma &= \langle P \rangle \pi a^2 \\ \langle P \rangle &= (Z_1 Z_2 \alpha)^2 (\alpha/8\pi)^2 (m_{\pi L}/f_{\pi L})^2 X \\ X &= \left\langle \int |U_0|^2 (d^3p/2Em_{\pi L}^2) \right\rangle \end{aligned} \tag{23}$$

where $\langle P \rangle$ is the average production probability obtained as the average over the impact parameters $b \leq a$. We have extracted the dependence of U on the nuclear charges so that U_0 corresponds to the production amplitude associated with a unit nuclear charge, for which Coulomb potential is $V = 1/r$.

Substituting the typical values of the nuclear charges and the estimate for f_{π_L}, one obtains for the production cross section the estimate

$$\sigma \simeq X \cdot 10^{-33}\ \text{m}^2 \tag{24}$$

The experimental estimates for the production cross section are of the order of magnitude $\sigma \simeq 10^{-32}$ m^2 (Tsertos *et al.*, 1985), so that the *correct order of magnitude* is obtained, since numerical evaluation of X (see Appendix) gives $X = 8.36 \pm 0.48$.

(e) The exchange of the leptopion implies a *new weak interaction between leptons*. This interaction couples right- and left-handed leptons to each other and its strength is of the same order of magnitude as the strength of the ordinary weak interaction at energies not considerably larger than the mass of the leptopion, which is at cm energies not much larger than 1 MeV. At high energies this interaction is negligible and the existence of the leptopion predicts no corrections to the parameters of the standard model, since these are determined from weak interactions at much higher energies.

If the *leptopion mass is sufficiently small* (as found, this is allowed by experimental data, $2m_e$), the interaction mediated by leptopion exchange can become quite *strong* due to the presence of the leptopion propagator.

3. HOMESTAKE–KAMIOKANDE PUZZLE AND LEPTOPION HYPOTHESIS

Suppose that the left-handed neutrinos produced in the solar core are transformed partially to right-handed neutrinos at solar spots by some mechanism to be considered in a separate paper. It is indeed possible that the leptopion hypothesis *explains the Homestake–Kamiokande puzzle* (Davis *et al.*, 1988; Hirata *et al.*, 1986) (which need not be actual, given the large uncertainties related to the Kamiokande measurements).

(a) The energies of solar neutrinos observed in Kamiokande are of the order of 5 MeV and their detection is based on their scattering from electrons, which are at rest on the average. The velocity of the electrons in the cm frame is given by $v = 1/(1 + me/E_\nu) \simeq 10/11$ and the center-of-mass energy of the electrons is $E_{cm} = m_e/(1 - v^2)^{1/2} \simeq \sqrt{3}\, m_e \simeq 0.9$ MeV. The center-of-mass energy squared is given by $s/me^2 \simeq 26.4$.

(b) If the mass of the lowest leptopion is sufficiently small, the interaction mediated by the leptopion exchange can be comparable to or even larger than the strength of the ordinary interaction and it is possible that right-handed neutrinos are detected in Kamiokande via the reaction $\nu_R + e_{L/R} \rightarrow \nu_L + e_{R/L}$ (see Figure 1). If the cross sections for the scattering of right- and left-handed neutrinos on electrons have comparable sizes $\sigma_R \simeq \sigma_L$, *the anticorrelation with sunspots should be weaker* for the effective flux measured *in Kamiokande* than for the left-handed neutrino flux observed

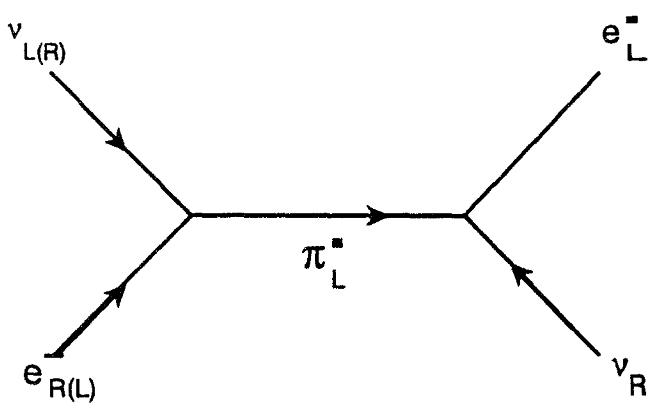

Fig. 1. Detection of neutrinos by leptopion exchange.

in Homestake by nucleon–lepton interactions. Observe that complete leptopion dominance implies identical cross sections for right- and left-handed neutrinos and the total vanishing of anticorrelation, but predicts too large an effective flux.

(c) A potential difficulty of models trying to solve solar neutrino problems assuming that neutrinos have a magnetic moment is related to supernova physics. The magnetic moment of the neutrino implies a considerable chirality flip rate for the left-handed neutrinos produced in supernova. If right-handed neutrinos are inert, they escape the supernova immediately, so that the neutrino burst from the supernova becomes shorter. The data obtained from SN1987A (Hirata *et al.*, 1987) are in accordance with the absence of right-handed neutrinos. *In the present case this problem is absent.* The reason is that the temperature inside a supernova is so high (hundreds of MeVs) as compared to the mass of the leptopion that the *rate for the chirality flip by leptopion exchange is negligibly small.*

(d) To see whether the mechanism indeed works, one must compare the *total cross sections for the scattering of left- and right-handed neutrinos on electrons.* The contribution of the standard electroweak interactions to the scattering $\nu_L^1 e_2 \to \nu_L^4 e_3$ (Okun, 1982) is given by the expression

$$\begin{aligned} T_{\text{stand}} &= (G/\sqrt{2})\bar{e}_2\gamma_\alpha[(1+\gamma_5)g_L + ((1-\gamma_5)g_R)]e_3\bar{\nu}_1\gamma \\ g_L &= 1/2 + \sin^2\theta_W \\ g_R &= \sin^2\theta_W \end{aligned} \tag{25}$$

The contribution of the leptopion exchange is given by the expression

$$T_{\pi_L} = (G/\sqrt{2})\bar{\nu}_1(\gamma_5 - 1)e_3[m_e^2/(q^2 - m_{\pi L}^2)]\bar{e}_2\gamma_5\nu_4 \tag{26}$$

The scattering amplitude $\nu_R e \to \nu_L e$ is essentially of the same form as the leptopion contribution to the scattering of ν_L.

The square of the total scattering amplitude is given by

$$\begin{aligned} |T_L|^2 &= |T_{\text{stand}}|^2 + |T_{\pi_L}|^2 + 2Re(T_{\text{stand}}T_{\pi_L}^\dagger) \\ |T_{\text{stand}}|^2 &= K[(g_L^2 + g_R^2)(p_1\cdot p_2)^2 + (p_1\cdot p_3)^2 - 4g_R g_L m_e^2 p_1\cdot p_4] \\ |T_{\pi_L}|^2 &= (K/2)\delta^2(p_1\cdot p_2)^2[m_e^2/(q^2 - m_{\pi_L}^2)]^2 \\ 2Re(T_{\text{stand}}T_{\pi_L}^\dagger) &= K\delta[g_L p_1\cdot p_4 m^2 - 2g_R(p_1\cdot p_3)^2]m_e^2/(q^2 - m_{\pi_L}^2) \\ K &= 8G^2/m_e^2 \end{aligned} \tag{27}$$

For future convenience we have included the parameter δ to measure the possible deviation of the leptopion coupling from the value predicted by the previous PCAC argument: $g = \delta \cdot g_{\text{PCAC}}$.

The ratio of the left- and right-handed total cross sections is given by the expression

$$\sigma_L/\sigma_R = \sigma_L^{\text{stand}}/\sigma_R + 1 + \sigma_{\text{int}}/\sigma_R \tag{28}$$

where the first term corresponds to the ratio of the standard model cross section to the right-handed cross section as predicted by TGD (Pitkänen, 1990*b*). σ_{int} results as an interference term between the standard model scattering amplitude and the leptopion exchange amplitude. The approximate expressions for the various quantities are given by

$$\begin{aligned} \sigma_L^{\text{stand}}/\sigma_R &\simeq \delta^{-4}(8r^3/3x)/\{1+[1-\ln(4x/r)](r/x)\} \\ \sigma_{\text{int}}/\sigma_R &\simeq -\delta^{-2}(20/27)[\ln(4x/r)/2+2g_R-g_L] \\ r &= (m_{\pi_L}/m_e)^2 \\ x &= s/m_e^2 \end{aligned} \tag{29}$$

It is essential that the interference term σ_{int} is negative, so that it in fact cancels the leptopion contribution $\sigma_{\pi_L} = \sigma_R$ to $O(10^{-1})$. Note that the ratio $\sigma_L^{\text{stand}}/\sigma_R$ is very sensitive to the value of the leptopion mass and the value of the leptopion coupling.

(e) To find the conditions on the parameters r and δ needed to explain the Kamiokande–Homestake puzzle, let us recall basic experimental facts about solar neutrinos.

(i) The average value of the neutrino flux measured in Homestake is about one-third the predicted value: $\Phi_{\text{Home}}/\Phi_{\text{pred}} \simeq 1/3$ (Davis, 1986).

(ii) The average value of the flux observed in Kamiokande is about one-half of the predicted value: $\Phi_{\text{Kamio}}/\Phi_{\text{pred}} \simeq 1/2$ (Hirata *et al.*, 1986). The neutrinos observed in Kamiokande have laboratory energies between 5 and 10 MeV. If our proposal is correct, then what is measured in Kamiokande is the following effective flux:

$$\begin{aligned} \Phi_{\text{Kamio}}/\Phi_{\text{pred}} &= (\Phi_L/\Phi_{\text{pred}})(\sigma_L/\sigma_L^{\text{stand}}) + (\Phi_R/\Phi_L^{\text{stand}})(\sigma_R/\sigma_L^{\text{stand}}) \\ \Phi_L + \Phi_R &= \Phi_{\text{pred}} \end{aligned} \tag{30}$$

(iii) Substituting the Homestake value for the average flux of left-handed neutrinos and requiring that the average value of the Kamiokande flux is about 1/2, one obtains an upper bound for the size of the right-handed cross section

$$\sigma_R/\sigma_L^{\text{stand}} \simeq 1/[4+2(1+\sigma_{\text{int}}/\sigma_R)] \leq 1/4 \tag{31}$$

Using the explicit expressions for the various terms appearing in the formula, one obtains conditions on the possible values of the parameters r and δ.

(e) Consider now the predictions for $\sigma_L^{\rm stand}/\sigma_R$ and effective neutrino flux at the *endpoints of the neutrino energy range*, assuming that the mass of the leptopion is $m_{\pi_L} \simeq 1.062$ MeV.

1. For $E_\nu = 5$ MeV and $\delta = 1$ one obtains $\sigma_L^{\rm stand}/\sigma_R \simeq 11.2$ and $\Phi_{\rm Kamio}/\Phi_{\rm stand} \simeq 0.43 < 0.5$. If one poses the requirement $\Phi_{\rm Kamio}/\Phi_{\rm stand} \simeq 0.5$, one obtains for the parameter δ the estimate $\delta \simeq 1.05$. The relative deviation from the predicted value is of the same order of magnitude as the relative deviation of f_π as predicted from the Goldberger–Treiman relation. The same effect is achieved if the mass of the leptopion is $m_{\pi_L} \simeq 1.093$ MeV instead of 1.062 MeV.

2. For $E_\nu = 10$ MeV and $\delta = 1$ one has $\sigma_L^{\rm stand}/\sigma_R \simeq 5.5$ and $\Phi_{\rm Kamio}/\Phi_{\rm stand} \simeq 0.45 < 0.5$. For $\delta = 1.05$ one obtains $\sigma_L^{\rm stand}/\sigma_R \simeq 4.4$ and $\Phi_{\rm Kamio}/\Phi_{\rm stand} \simeq 0.55 > 0.5$.

The neutrino flux observed in Kamiokande is predicted to *anticorrelate with sunspots*, although the *range of the variation is smaller* than in the case of Homestake. The range of the variation for $\Phi_{\rm Home}/\Phi_{\rm pred}$ is (0.25, 0.8) and implies that the range of $\Phi_{\rm Kam}/\Phi_{\rm stand}$ is approximately given by (0.35, 0.85) for $E_\nu = 5$ MeV and (0.42, 0.85) for $E_\nu = 10$ MeV.

To summarize, the leptopion hypothesis provides an explanation for the Homestake–Kamiokande problem provided part of the solar neutrinos are transformed to right-handed neutrinos in the convective zone of the Sun and the transformation rate correlates with the sunspots. We leave the description of the TGD-inspired model for the $\nu_L \to \nu_R$ transformation in a forthcoming paper.

APPENDIX. NUMERICAL ESTIMATE FOR PRODUCTION CROSS SECTION

The total production cross section for leptopion production can be evaluated numerically. The production amplitude possesses a singularity, but since we are using the average production probability, it is possible to get a numerical estimate. The most important expression is U in equation (21). The expression involves the Fourier transforms of the fields E and B given by the expressions (Pitkänen, 1990*a*)

$$\begin{aligned} E_i(k) &= \delta(k_0)k_i/k^2 \\ B_i(k) &= \delta(\gamma(k_0 - \beta k_z))k_j\varepsilon_{ijz}\, e^{ik_x b}/[(k_z/\gamma)^2 + k_T^2] \end{aligned} \tag{A1}$$

where the coordinates are chosen so that the target nucleus is at rest at the origin of coordinates and the colliding nucleus moves along the positive z direction in the $y = 0$ plane with velocity β.

The expression for the amplitude reduces to the following general form:

$$U = CP_1 + CP_2 + RP \tag{A2}$$

where RP is real plane contribution of the poles to the amplitude. $CP_{1,2}$ are the contributions by two cuts.

The expression for the pole contribution RP is

$$RP = e^{ik\eta}\left\{\sin\theta\sin\phi - i[1+(k\beta\gamma)^2]^{1/2}\left(\pi\sin\theta\cos\phi\,\frac{q(n^2+q^2\sin^2\theta)^{1/2}}{2}\right)\right.$$
$$\left. + [n^2 + (k - v\sin\theta\cos\phi)^2]^{1/2}\right\} \tag{A3}$$

where we have used following notations:

$$\begin{aligned} n &= 1 - q\cos\theta \\ \gamma &= \frac{1}{(1-\beta^2)^{1/2}} \\ q &= v_{\rm cm}/\beta \\ v_{\rm cm} &= \frac{2v}{1+v^2} \\ \eta &= bm\gamma_1 \end{aligned} \tag{A4}$$

The expressions for cuts $CP_{1,2}$ contain definite integrals which must be carried out numerically. These contributions are singular for certain values of kinematical variables (Pitkänen, 1990*a*). The explicit expression for the contribution of these terms reads

$$CP_1 + CP_2 = \sin\theta\sin\phi\int_0^{\pi/2} C_1\,\frac{1}{2}\,d\xi$$
$$+ (u\sin\theta\sin\phi)\,e^{i\eta u\sin\theta\cos\phi}\,\frac{1}{2}\int_0^{\pi/2} C_2\,d\xi \tag{A5}$$

where we have made a change of the integration variable ($\xi \in [0, \pi/2]$) and where various auxiliary variables are defined as

$$C_1 = \frac{e^{\eta \cos\xi}(\sin\theta\cos\psi + iG\cos\xi)}{R_1}$$

$$xC_2 = e^{\eta\mu\cos\xi/\beta}\left[\sin\theta\cos\phi u + i\cos\xi\left(\frac{n}{v_{\rm cm}} + \frac{v}{\beta}\sin^2\theta(\sin^2\psi - \cos^2\psi)\right)\right]\Big/R_2$$

$$R_1 = \sin^2\theta(\sin^2\theta - \cos\xi) + G^2 - 2iG\cos\xi\sin\theta\cos\psi$$

$$R_2 = \sin^2\theta\left[\frac{v^2\sin^2\theta}{\gamma^2} - u^2\cos^2\xi + \beta^2\cos^2\phi\left(-\frac{2vn}{v_{\rm cm}} + v^2\sin^2\theta\right)\right] \tag{A6}$$

$$+\left(\frac{v_{\rm cm}}{n}\right)^{-2} + 2iu\beta\sin\theta\cos\phi\left(v\sin^2\theta\cos\phi - w\frac{\cos\xi}{v_{\rm cm}}\right)$$

$$G = \beta\gamma\left(1 - v_{\rm cm}\frac{\cos\theta}{\beta}\right)$$

$$u = 1 - \beta v\cos\theta$$

For numerical integration we have used a VAX-8800-based Macsyma symbolic calculation program and i836-based Mathematica with self-made integration routine [following Press *et al.* (1989)]. Since we first evaluated analytically the differential cross section, it was possible to write a code which notices the singular effect. Substituting the typical nuclear charges and estimates for f_{π_L} into the numerical program, we found the production cross section to be

$$\sigma \simeq X \cdot 10^{-33}\ \mathrm{m}^2 \tag{A7}$$

where $X = 8.36 \pm 0.48$. The variance depends on the estimates used; the average accuracy at integration is better than 0.2%, which is accurate enough for our estimates.

ACKNOWLEDGMENTS

We thank J. Maalampi and J. Peltoniemi for enlightening discussions concerning various aspects of the solar neutrino problem.

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